A Model to Predict the Maximum Car Speed in the Rain

Chen Mingming, Chen Shiyi, Gong Liancheng,
Tang Xiaoyan, Yuan Shuhan

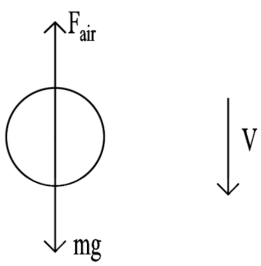
January 3, 2021

Abstract

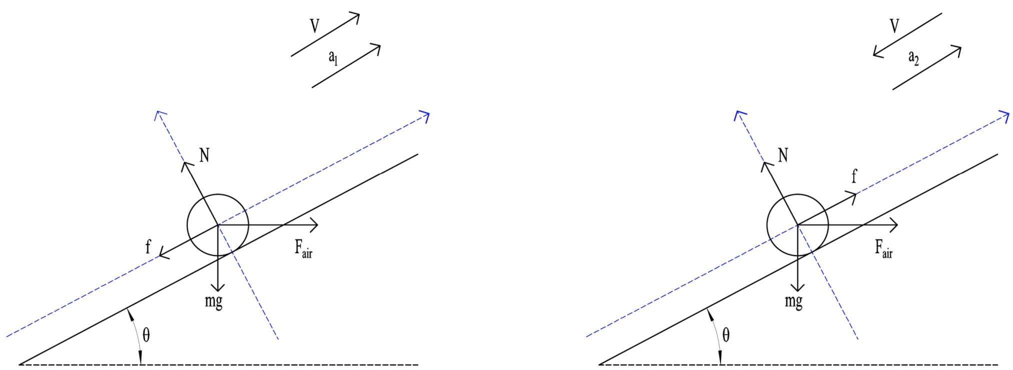
The speed that a car can maintain on rainy days depends on many factors. This paper will first introduce a model to quantify the visibility of the driver using the ratio between the surface area of all raindrops on windshield and the area of the windshield. In addition to driver's comfortability of visibility, we also take the sudden brake, sleep-deprived driving, road condition and light change, into the safe speed consideration. In this paper, we build a model to compute the safe speed of the car under real-life considerations by combining mathematical and physical knowledge to provide a concrete and useful insight into this problem.

keywords: traffic safety, math modeling, simulation

1 Preliminary

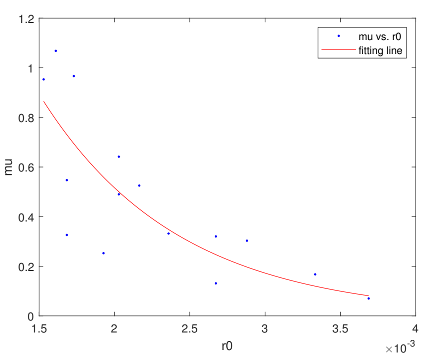


(a) Force analysis for falling rain drop

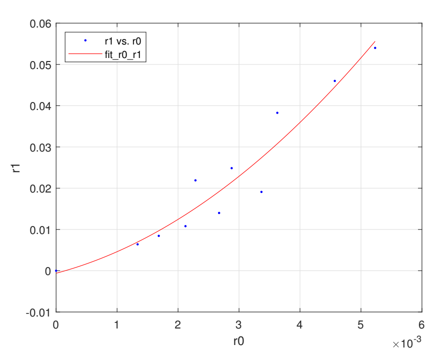


(b) Force analysis for rain drop on the window

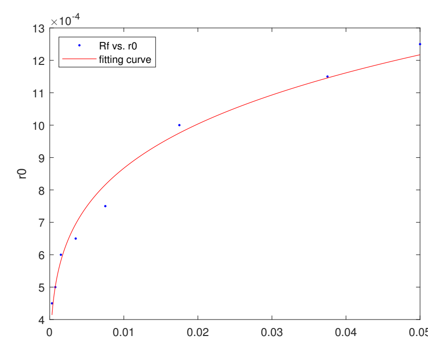
Figure 1: Force analysis



(a) Experiment result for μ and r0 and the regression line



(b) Experiment result for r1 and r0 and the regression line



(c) Statistical relationship on r0 and Rainfall[1]

Figure 2: Relationships

Name of the variable	Description
ρ_{water}	the density of water
ρ_{air}	the density of air
θ	the angle between the window and the horizontal line
l	the length of the glass
μ_1	the dynamic friction factor of the raindrop on the window
v_0	the velocity of the raindrop before dropping on the windshield
r_0	the radius of the raindrop in the air
r_1	the radius of the raindrop on the windshield
S	sum of the surface area of the raindrop on the windows surface
g	the acceleration of gravity
L	the available distance between the truck and Xiaoming's car
Ss	the visibility $:= \frac{S}{S_{\text{wiper}}}$
V	the volume of the raindrops
Rf	the rainfall
N	the upward support of the window to the raindrop
f	the friction between the raindrop and the window

Table 1: Notations used in this report

1.1 Knowledge

1.1.1 The measurement of the rainfall intensity

Rainfall intensity is classified according to the rate of precipitation in unit time. The following categories are used to classify rainfall intensity:
Light rain: precipitation rate <2.5 mm per hour
Moderate rain: precipitation rate between 2.5 mm to 10 mm per hour.
Heavy rain: precipitation rate >between 10 mm and 50 mm per hour.
Violent rain: precipitation rate >50 mm per hour.

1.1.2 Relationships

Relationship between R_f and r_0 According to the data from Yu et al[1], the relationship between R_f and r_0 is statistically shown in Figure 1 (c), we plot the red line as the regression function.

Experiment for the relationship between r_0 and r_1 We take a syringe to drip a water drop onto the glass. The difference between the number read from the syringe is the volume of the water drop, denoted as V.
So we have:

$$V = \frac{4}{3}\pi r_0^3$$

Then we spread out the water drop on the glass to the critical situation that the little pool of water will not shrink or expand and use an electronic vernier caliper to measure the radius of the water on the glass, which is r_1.
Then we can use MATLAB curve fitting Tool box to plot (r_0, r_1).

Experiment for the relationship between r_0 and μ_1 Fix the glass and denote the height as h. We take a syringe to drop water on the glass. We drop the water until the critical situation when the water starts to slide down along the glass. The difference between the number read from the syringe is the volume of the water drop, denoted as V.
So we have:

$$V = \frac{4}{3}\pi r_0^3$$

Then we use the Newton's second law to get the equation:

$$\mu_1 mgcos(\theta) = mgsin(\theta)$$

$$\mu_1 = tan(\theta) = \frac{h}{\sqrt{l^2 - h^2}}$$

Because the critical situation and μ_1 are relevant to h, we can use Matlab curve fitting Tool box to plot:

2 Visibility Model

2.1 Rainer

In this section, we do force analysis of raindrops in the air to model the number of raindrops dropping down on the windshield. Two forces acting on the raindrop are gravity and air resistance. At the beginning, gravity is greater than air resistance and leading to the raindrop's speed up. With the increase of the velocity, air resistance will increase to the same as the value of gravity. In final stage, all forces balances and the raindrop does a uniform linear motion. Thus, we can calculate the velocity of raindrops by solving the following equation using the resistance formula of the object in fluid motion:

$$\frac{1}{2}\rho_{air} C_d S v^2 = mg \quad (1)$$

$$S = \pi r_0^2 \quad (2)$$

$$mg = \rho_{water} V g = \rho_{water} \frac{4}{3}\pi r_0^3 g \quad (3)$$

After calculating the above 3 equations, we will have:

$$v_0 = \sqrt{\frac{\rho_{water}\frac{4}{3}\pi r_0^3 g}{\frac{1}{2}\rho_{air} C_d \pi r_0^2}} = \sqrt{\frac{8\rho_{water} r_0 g}{3\rho air C_d}} \quad (4)$$

Next, we will calculate the volume of the raindrops. The raindrops dropped on the windshield are divided into two parts – vertical drops and horizontal drops. Equations are the following:

$$V_{vertical} = RFdtS_1 = RFdtwin_1win_2\cos\theta \tag{5}$$

$$V_{horizontal} = \frac{RFdtv_{car}win_1win_2\sin\theta}{v_{rain}} \tag{6}$$

$$V_{totalrain} = V_{vertical} + V_{horizontal} \tag{7}$$

We can understand equation (6) by regarding $(dtv_{car}win_1)$ as the bottom area of the formed space and regarding $\frac{RFwin2\sin\theta}{v_{rain}}$ as the thickness of raindrops per area. By summing up equation (5) and (6), we will get the total volume of raindrops on the windshield. Considering the non-identical radius of raindrops, we follow the normal distribution to calculate radius.

2.2 mover

In this section, we do force analysis of raindrops on the windshield to model the acceleration and update the speed and position of each raindrop. Four forces acting on the raindrop include gravity (mg), air resistance (F_{air}), normal force (N), and friction (f) as shown in the above graph. Gravity is vertical downward, the normal force is perpendicular outward to the windshield, air resistance is horizontal rightward assuming that car is moving leftward, friction is either upward or downward along the windshield depending on moving direction. Considering a new coordinate system.

Assuming that the raindrop moving upward along the windshield:

$$N = mg\cos\theta + F_{air}\sin\theta \tag{8}$$

$$f = N\mu \tag{9}$$

$$F_{air}\cos\theta - mg\sin\theta - f = ma \tag{10}$$

Combining equation (8), (9), (10), we will get acceleration a:

$$a_{upward} = -g\cos\theta\mu - g\sin\theta + \frac{3C_d\rho_{air}v^2}{8\rho_{water}r}(\cos\theta - \sin\theta\mu) \tag{11}$$

Same as raindrop moving downward along the windshield:

$$a_{downward} = g\cos\theta\mu - g\sin\theta + \frac{3C_d\rho_{air}v^2}{8\rho_{water}r}(\cos\theta + \sin\theta\mu) \tag{12}$$

Then, based on the former direction of the raindrop, we may apply either $a_{downward}$ or a_{upward}. However, if previous velocity of the raindrop is 0, which means that it has just dropped down onto the windshield. Recalling our new coordinate system, if both $a_{downward}$ and a_{upward} are negative, the raindrop must move downwards along the windshield, which means that we should apply $a_{downward}$. Meanwhile, if both $a_{downward}$ and a_{upward} are positive, the raindrop must move upwards along the windshield, which means that we should apply a_{upward}. With the previous velocity and the acceleration rate, we can now update the current velocity with equation $v_t = v_0 + at$ and update current y coordinate with equation $y_t = y_0 + v_t t$.

2.3 wiper

In this section, we model the wiper's motion and calculate the location of the wiper. If the wiper is moving forward then the updated angle is $(\theta_0 + \frac{2\pi dt}{t_1})$; meanwhile, if the wiper is moving backward then the updated angle is $(\theta_0 - \frac{2\pi dt}{t_1})$.

2.4 simulator

In this section, we combine previous rainer, mover, and wiper models to calculate the visibility through the windshield.

We calculate the raindrops wiped by the wiper and the raindrops within the driver's vision. Firstly, we filter all the raindrops that remain in the windshield by comparing y coordinate with the windshield's position after the mover model. Then, we calculate the distance and angle between each raindrop and the center of the wiper. By comparing distance and the wiper length, we can filter the raindrops within the range of the wiper. By comparing the angle and the location of the wiper modeled by the wiper, we can filter and clear those raindrops wiped by the wiper. Thus, we may now update the new rain list into the current raindrops within the driver's vision.

After filtering the raindrops, we can calculate the visibility through the following equation:

$$Ss = \frac{sum(\pi r^2)}{\frac{1}{2}\pi(\frac{win_1}{2})^2} \tag{13}$$

3 Real-life Case Model

In the above sections, we introduced a very powerful visibility model that can help us maintain the comfortable visibility of Xiaoming during his driving. However, in real life, the safe speed limit should not only constrained by the driver's personal visibility feeling. We should also take other safety facts into consideration. In this section, we will discuss in detail that how other real-life scenarios will influence the driver's safe speed limit by taking the following four factors into consideration: 1) unprepared sudden brake of the front truck 2) sleep-deprived driving 3) different friction coefficient between vehicle tire and road surface resulting from road conditions 4) visibility change due to different time in a day.

3.1 front truck sudden brake

It is often the case that in real-life, you will driving behind a truck and has a limited field of view. So you can't expect whether the front traffic situation will lead to the brake of the truck. When it happens, it becomes a sudden brake.

In this and the following subsection, we will assume that the driver is facing a front truck has a sudden brake to speed 0 (which means the truck will remain in the same space) and the available braking distance between the driver and the front trunk is L. Suppose that the driver needs time t to realize the front trunk brake and start to brake his/her car and the friction coefficient between the vehicle tire and road surface is μ.

Assume the driver's driving speed is v. The driving distance during driver's reaction time is:

$$d_1 = vt$$

The deceleration during the brake only results from the friction between vehicle tire and road:

$$a = \mu g$$

The time takes the driver to brake to speed 0 is:

$$T = \frac{v}{a} = \frac{v}{\mu g} \tag{14}$$

The brake distance is:

$$d_2 = \frac{1}{2}aT^2 = \frac{1}{2}\frac{v^2}{\mu g} \tag{15}$$

The total brake distance that the driver need is:

$$D = d_1 + d_1 = vt + \frac{1}{2}\frac{v^2}{\mu g} \tag{16}$$

To ensure the safety, we need to have $D \le L$, then, we can have the safety speed limit:

$$v \le \mu g\sqrt{t^2 + \frac{2L}{\mu g}} - t\mu g \tag{17}$$

3.2 case of sleep-deprived driving

In this section, we will introduce the sleep-deprived situation and measure the danger of sleep-deprived driving by comparing the specific safe speed limit difference in normal driving and sleep-deprived driving in one concrete circumstance.

According to the result from a visual choice reaction time task[6] the average reaction time in normal situation is around 0.5 second. However, the average time of a yawn is about 6 second. So if we assume that we have a $100m$ brake distance and the friction coefficient is 0.5, the gravitational acceleration is $10ms^{-1}$, than after a round computation, we can have that:

$$v_{normal} \approx 100kmh^{-1}, v_{sleepy} \approx 48kmh^{-1}$$

So in this case study, we do verify that whether the driver is sleepy or not exerts significant impacts on the safe speed limit.

3.3 road condition-dependent friction coefficient change

During raining days, one important factor that will greatly influence our safe speed limit is the condition of the road surface. According to several studies, the friction coefficient on dry load may range from $\mu_{dry} = 0.5$ [5] to $\mu_{dry} = 1.0$ [3], and the friction coefficient on wet road may range from $\mu_{wet} = 0.2$ [5] to $\mu_{wet} = 0.5$ [3]. Generally, the friction coefficient of dry road will be 2 times of that of rain road and we will use such a relationship in the following calculation. Assume in such a condition that it suddenly rains during Xiaoming's driving, the safe speed limit before raining is:

$$v_{dry} \le \mu_{dry} g\sqrt{t^2 + \frac{2L}{\mu_{dry} g}} - t\mu_{dry} g \tag{18}$$

and the safe speed limit when it rains is:

$$v_{wet} \le \mu_{wet} g\sqrt{t^2 + \frac{2L}{\mu_{wet} g}} - t\mu_{wet} g \tag{19}$$

We use the relationship between μ_{dry} and μ_{wet} by replacing the μ_{wet} in the above equation by $\mu_{wet} = \frac{\mu_{dry}}{2}$. Since t is significantly smaller than the other terms (t will be on the order of 1/5 of the other terms), we ignore t in the calculation of the ratio of safe speed limit on a dry road to that on a lubricated road. And we can a simplified version of equation (19):

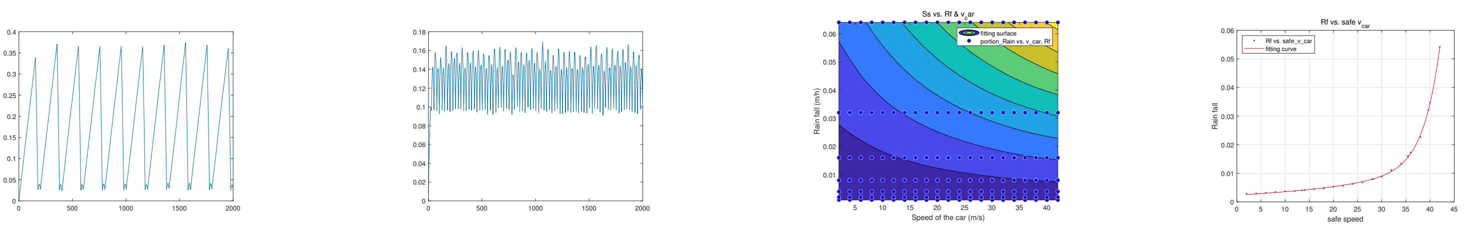
$$v_{limit} = \mu g\sqrt{\frac{2L}{\mu g}} \tag{20}$$

Therefore, we can have the ratio of the limit safe speed on between a dry road to a wet road:

$$v_{wet\_limit} = \frac{v_{dry\_limit}}{\sqrt{2}} \tag{21}$$

3.4 time-dependent visibility change

Another factor affecting limit speed for safe driving is the ambient lighting. Here we use a compensation model, in which the negative effect of lighting need to be compensated by decreased speed. We use data from a study on the relation between road lighting and accident rate [2]: for every $0.5cd/m^2$ increases average luminance at night, crash rate drops 19%. For the relation between speed and accident rate, based on the study by Nilsson[4], and for the sake of simplicity, we will adopt per 25 % increase in speed limit leading to 20 % more accidents. Therefore, we conclude that each $0.5cd/m^2$ decrease in average iluminance at night need to be compensated by 25.7 % increase in safe speed limit.



(a) When speed equal to 0, Ss on the window

(b) when speed equal to 30, Ss on the window

(c) Relationship between Ss vs Rf and v car

(d) Relationship between Rf and safe speed of v Car

Figure 3: Simulation Result

4 Result for Visibility Model

We simulate different with different speed and different rainfall to get the Figure 3.

The change of Ss under certain v_{wiper} and v_{car}. Figure 3(a)(b) depict how Ss changes under certain v_{car} and v_{wiper} change with time measure in millisecond.

The relationship among Ss, Rf and v_{car} The blue points shown in the figures are the simulation result. After the simulation, we use the 3D curve fitting to fit the blue points. Contour lines describe a surface in a 3-D coordinate system, with v_{car} on the X-axis, Rf on the Y-axis, Ss on the Z-axis as shown in Figure 3(c).

The Situation When Xiaoming is Comfortable to Start Driving The project states that Xiaoming is comfortable to start driving when the wiper works at once per two seconds. So we set $v_{wiper} = 0.5$, $v_{car} = 0$, then we plot another 3-D surface of Ss, Rf and v_{car}. Then the intersection of the two surfaces would be the line of Rf and v_{car} under the Ss when Xiaoming is comfortable to drive as shown in Figure 3(d).

5 Conclusion

In conclusion, the highest comfortable driving speed for Xiaoming can be estimated given the initial condition, the current precipitation rate and front windshield angle. However, this maximum comfortable speed as shown in the second plot of Figure 3(d) may not be the only safe speed limit for Xiaoming. The actual safe speed limit should be estimated by consider all of the real-life scenarios discussed in section 2. The code for visibility model can be found at: https://github.com/Eleven7825/Rainfall_simulator. The video for simulation can be found at http://eleven.nyushers.org/intro-to-math-modeling-final-project/.

6 Limitation and future work

The experiment to find the relationship between r_0 and r_1 We only use a glass in the static state to simulate the statement for the car running on the highway. This may cause some error for our experiment data.

The radius of the raindrop In our visibility model, we assume the radius of raindrops follow the normal distribution. From the paper from Yu Ma et al.[1] the distribution of the diameters in different rainfall may not be normal, so our approximation may have some shortcomings.

The Future Application of This Work In this model, we combined the driver's driving condition and driver's personal driving preferences to decide the safe speed. The business application of this research can be used to construct an app to compute the safe speed for the driver. The app will remind user not to drive when sleepy and automatically compute the safe speed based on the current light condition and weather data. Combining with GPS, we may also let the app set out an alarm to let the driver know.

7 Work division

Chen Shiyi: experiments, propose model, coding, drew Matlab curve; Tang Xiaoyan wrote abstract, preliminary, result, restriction and future application of the final paper, check visibility model.
Gong Liancheng: wrote Section 2 Model part of the final paper, check visibility model.
Chen Mingming and Yuan Shuhan: propose Real-life Case Model, check visibility model.
All five, check and polish the report.

References

[1] Baojun Chen, Jun Yang, and Jiangping Pu. Statistical characteristics of raindrop size distribution in the meiyu season observed in eastern china. *Journal of the Meteorological Society of Japan. Ser. II*, 91(2):215–227, 2013.

[2] Michael Jackett and William Frith. Quantifying the impact of road lighting on road safety—a new zealand study. *IATSS research*, 36(2):139–145, 2013.

[3] Steffen Mu¨ ller, Michael Uchanski, and Karl Hedrick. Estimation of the maximum tire-road friction coefficient. *J. Dyn. Sys., Meas., Control*, 125(4):607–617, 2003.

[4] Anders Nilsson. Spatial differentiation of ectoparasites on small mammals. *Ecography*, 4(3):184–190, 1981.

[5] WJ Wang, Peng Shen, JH Song, J Guo, QY Liu, and XS Jin. Experimental study on adhesion behavior of wheel/rail under dry and water conditions. *Wear*, 271(9-10):2699–2705, 2011.

[6] David L Woods, John M Wyma, E William Yund, Timothy J Herron, and Bruce Reed. Age-related slowing of response selection and production in a visual choice reaction time task. *Frontiers in human neuroscience*, 9:193, 2015.