Measurement of rotational torque caused by a dissolving object

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Abstract

Our project measures the change of torque caused by a hard candy ball of radius 2.4 cm. According to our measurement, the torque the dissolving ball received from the fluid can reach 0.1 dyne·cm during the whole process of dissolution. Along with the trend over in a smaller time scale, fluctuation in the torque with amplitude around 0.01 dyne·cm and period about 30 - 40 seconds is also observed. Also, by increasing the sugar concentration in the background fluid, the carrying fluctuation's amplitude decreases.

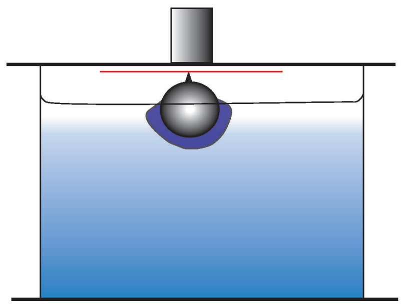
1 Introduction

The shape of the dissolvable body could be altered due to the existence of a nearby flow structure. The surrounding fluid-flow could be unidirectional flows[2], the gravity-induced flow[3]. While the shape dynamics of the dissolvable body have been modeled and studied, our understanding of the mechanism of how fluid exerts forces on the dissolvable body is limited.

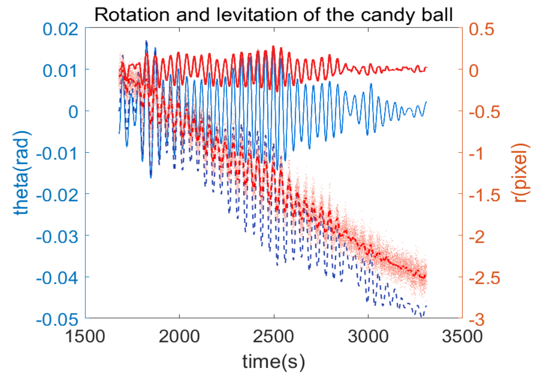
Recent research shows that melting ice disk can generate a vertex under the disk, this vortex then entrains the ice disk to have a rotational movement[1]. Our recent experiments also show indications for this rotational movement in our floating candy setup. Figure 1a shows our floating candy experiment, a candy ball containing a hollow stainless steel ball is placed into the degassed-deionized water. The candy ball is eccentric so that it has a preferred orientation when floating on the water. We place two fluorescent lights(not showing) at two ends of the bar(appearing red in the figure) for image tracking. A neodymium magnet is placed on the acrylic board so that it centers the candy ball in the middle of the container. A camera is fixed on a tripod above the floating candy ball and records the movement of fluorescent lights(not showing in Figure 1a). The results are shown in Figure 1b, red lines are the distance in which the center of two fluorescent lights move compared to time zero while the blue lines are change of orientation of the bar compared to time zero. The dotted lines are the actual data while the solid lines are the detrended data. From Figure 1b, not only the big trend of rotation can be seen, but also the fluctuation along with this trend can be observed.

The movement pattern shown in Figure 1b triggers our interests in the cause of the force account for the movement. How large is the force? Is there any vortex generated by the concentration plume of candy? How the movement could change if we try to increase the sugar concentration in the background fluid? To answer these questions, we designed another experiment to measure this force. Since Cavendish's method is successful in measuring small torque, we decide to adopt a similar method to measure the rotational torque caused by the dissolving candy.

In 1797, Henry Cavendish measures the gravitational force between two objects and thus give out the value for the gravitation constant. Figure 2 is the setup for Cavendish's experiment. Two identical balls A and B with mass m are connected with a rod. Balls A, B, and the bar are hanging by a wire OO'. A mirror is fixed at the end of the wire which reflects the light. The functionality of the mirror is to reflect the light and increase the limit of observation. Two larger balls C and D with mass M are placed beside A and B. By measuring the movement before and after C, D is near A, B, the torque caused by their gravitational forces can be calculated.



(a) Schematic setup of experiment



(b) Measured angle and relative distance

Figure 1: Floating candy experiment

2 Experiment setup

This section focuses on the setup and procedure of the experiment. A partial setup schematic can be found in Figure 3a. All apparatuses are fixed on the optical platform to minimize the vibrations from the ground. The suspension system is also covered with plastic wraps to prevent disturbance from the air(wraps and supporting materials not showing in the Figure). On the right side of Figure 3a, a cylindrical acrylics container encircles a thin mylar cylinder, their bottom are stick together using silicone glue. The inner cylinder serves as a separation layer so that we do not need to change the outer container if we want to change the circumference of the container. During the experiment, we keep the water level inside and outside of the inner mylar cylinder to be the same to minimize the pressure difference at the bottom of the two cylinders. By varying the circumference of the inner mylar cylinders(inner cylinder for future reference), the goal of changing the effective cross-section area is achieved. Three inner cylinders with different circumferences were made and tested in our current setup and the result will be included in Section 4. The inner cylinder has diameter D_1 while the outer cylinder has diameter $D_2 = 250$ mm.

The suspension part contains a candy, a beam, and a copper wire. We use screws for connections between the candy, the wire, and the bean. In the meanwhile, the wire and the screws are connected using electrical welding while other parts are tightened together through the threads of screws. The candy ball of radius 2.4 cm is fixed at the bottom of the copper wire, while the top of the wire connects a horizontal aluminum beam. The beam then connects to a vertical platform whose height is adjustable through an Arduino program. The platform can reach anywhere within the range. A glass slide with one side painted black sticks to the wire and serves as the mirror reflecting the incident light.

As for the light path. A laser light source fixing on the optical platform emits a light beam that hits the mirror on the wire. At the beginning of the experiment, we fix the incident light and adjust the screw which connects the copper wire and the beam such that the incident light hits the mirror and the reflected light hits in the region within 9.15 cm from the center of the screen. Figure 3b shows the light path of the laser from an overlook point of view. As for the measurement of the angle, more details will be provided in Section 3.2.

The lightpath setup mentioned above requires the angle between the principal axis and the reflected light to be small enough for us to apply the small-angle approximation. When the light spot lies within 9.15 cm from the main axis, we can consider the reflected light beam is perpendicular to the screen, as such, the

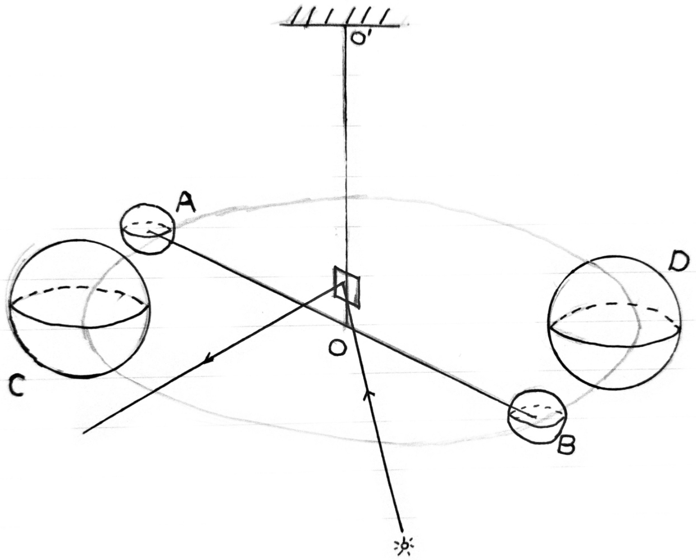
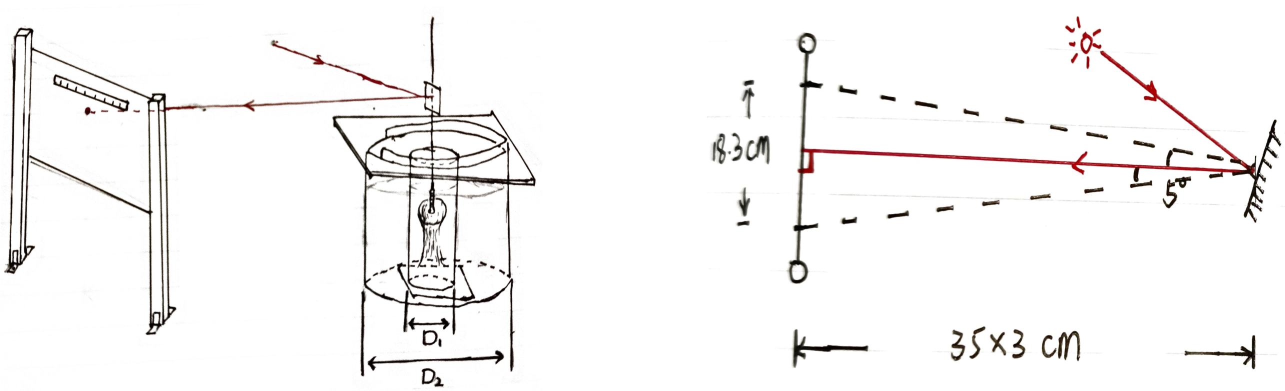


Figure 2: Cavendish's experiment

small-angle approximation applies. Behind the screen, a webcam that is also fixed on the tripod records the light spot on the screen. We track the change of the angle by tracking the location of the light spot.



(a) Three dimensional schematic

(b) Light path from overlook angle

Figure 3: Schematics of the experiment setup

3 Measurement Methods

3.1 Measurement of the torsion spring constant

This section focuses on how the change of the torque is measured based on the setup in Section 2. The Hook's law for the torsion spring tells us that,

$$\Delta\tau = -\kappa\Delta\alpha,$$

while κ is the torsion spring constant, $\Delta\alpha$ is change of the angle and $\Delta\tau$ is the change of torque given by the torsion spring. The period of a normal spring can be measured as

$$T = 2\pi\sqrt{\frac{m}{k}},$$

while m is the mass of the object on the spring.
Similar to the normal spring, the period of the torsion spring can be measured as

$$T = 2\pi\sqrt{\frac{I}{\kappa}},$$

while I is the moment of inertia of the object on the edge of the torsion spring and κ is the torsion constant of the torsion spring, rewrite the formula, we have,

$$\kappa = \frac{4\pi^2 I}{T^2}.$$

By simply measuring T of an object hanging in the still air and its corresponding moment of inertia, we could deduce the torsion spring constant. Plus the change of the angle during the dissolution, we could deduce the change of the torque.

We made a resin ball using the same silicon mold as we made the candy balls. We consider the mold as perfectly spherical, as such, we can use the formula

$$I = \frac{2}{5}mr^2$$

to calculate its moment of inertia of the resin ball as long as we know mass of the resin ball - m and the radius of the resin ball - r. And since we use the same mold, we know that the radius is 2.4 cm. Along with previous formulas, we can get the torsion spring constant κ. Take our most recent wire as example:

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2}{T^2}\frac{2mr^2}{5} = \frac{4\pi^2}{25.8^2}\frac{2\cdot 67\cdot 2.4^2}{5} = 9.15(\text{dyne}\cdot\text{cm/rad}).$$

Beside purely measurement, κ can also be deduced by some ideal model of torsion spring. As we know shear modulus G, torsion constant J, and the length of the wire L. Adopting shear modulus $G = 45\times 10^9$ for copper, $J = \frac{\pi}{2}r^4$ for circular cross-section, (the radius of the cross section$r = 0.05mm$). κ of the above wire can be calculated as,

$$\kappa = \frac{GJ}{L} = \frac{45\times 10^9 \times \frac{\pi}{2}(0.05\times 10^{-3})^4}{0.5}\text{N}\cdot\text{m/rad} = 8.83\times 10^{-7}\text{N}\cdot\text{m/rad} = 8.83(\text{dyne}\cdot\ \text{cm/rad}).$$

Although the length of our wire is 0.6 m, the effective length is shorter because there exists some distance between the mirror and the candy. That is the reason why we take $L = 0.5$ m instead of 0.6 m. As for comparison, the torsion spring constant for a wire with diameter $d = 0.5$ mm is around 3500 dyne $\cdot$ cm/rad. This comparison gives us the motivation to use thinner wires instead of thicker wires when it comes to small torque measurements.

3.2 Measurement of $\Delta\alpha$

Fixing the incident light beam, we can deduce the change of mirror angle($\Delta\alpha$) by measuring the distance in which the light spot moves as shown by Figure 3b. The angle change in reflected light $\Delta\theta$ is two times the angle change of the mirror $\Delta\alpha$, i.e.

$$\Delta\theta = 2\Delta\alpha.$$

Since our setup ensures the requirements for the small-angle approximation, we can calculate $\Delta\theta$ by

$$\Delta\theta = \frac{\Delta l}{M},$$

while M is the length of the main axis and Δl is the relative location compared to time 0. By tracking the location of the light spot at each time, we will be able to know $\Delta\theta$, moreover, $\Delta\alpha$ and the torque caused by the candy.

As for the algorithm used to calculate the location of the light spot, we used weighted sum to determined the center of the light spot. Firstly, we map three layer of red, green, blue value to only one layer of gray level. By representing the pixel values of each layer by r, g, b respectively, the gray value $\hat{g}$ can be calculated as

$$\hat{g} = \omega_1 r + \omega_2 g + \omega_3 b \quad \text{and} \quad \omega_1 + \omega_2 + \omega_3 = 1.$$

Since r, g, b are integers from 0 to 255, the gray level we get from this formula is also from 0 to 255. Secondly, we map the gray level from the first step to 0 to 1 using a sigmoid function. A sigmoid function in our case takes the form of

$$\omega = \frac{2}{1 + \exp(-\frac{\hat{g} - 255}{\sigma})}.$$

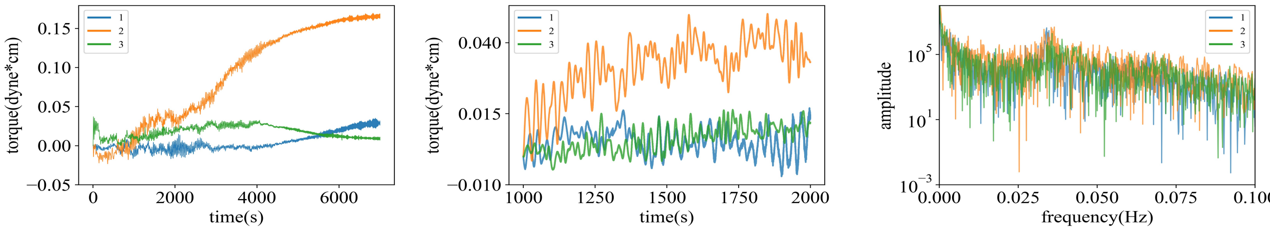
After these two steps, we can associate every pixel in the photo with a weight ω. Finally, we calculate the average location of all pixels in the photo with weight ω associate with them. The implementation in MATLAB is listed below:

```
function [x,y] = weightFit(img,sigma,rgb)
img = double(img);
f = @(X,sigma) 2./(1+exp(-(X-255)./sigma));
imgGray = (img(:,:,1).*rgb(1)+img(:,:,2).*rgb(2)+img(:,:,3).*rgb(3))./sum(rgb);
sz = size(imgGray);
weights = f(imgGray,sigma);
weights(weights<0.05) = 0;
[yy,xx] = meshgrid(1:sz(2),1:sz(1));
sumweights = sum(sum(weights));
x = sum(sum(xx.*weights))/sumweights;
y = sum(sum(yy.*weights))/sumweights;
end
```

4 Result

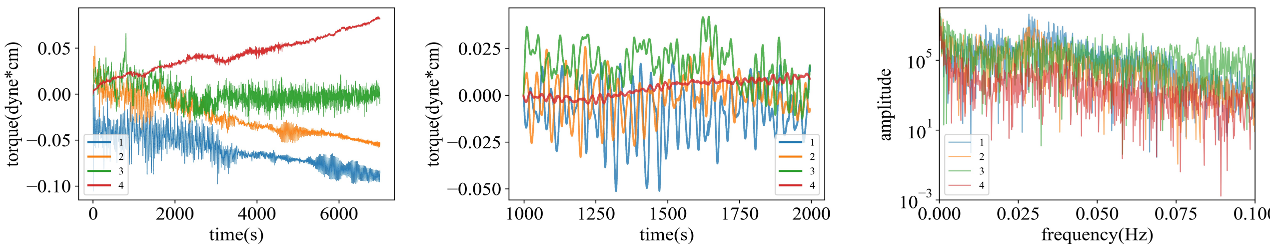
Since our hypothesis for rotational torque is based on the existence of the vortex under the dissolving candy, our experiments attempt to vary the conditions in which different vortexes may generate. As such, we try to change sugar concentration in the initial background fluid, the diameter of the inner cylinder to see if any changes corresponding to the torque may occur. Two groups of experiments are designed. The first group tries to controls the conditions for the experiment - all performed without the inner cylinder, no sugar in the initial background fluid. In contrast, the second group of experiments varies the parameters. We include four rounds in the second group of experiments. The first round adopts inner cylinder diameter $D_1 = 6.7$ cm with no sugar in the initial background fluid. Same as the first round, the second and the third rounds adopt D_1 to be 8.6 cm and 16.2 cm respectively. In the fourth round, $D_1 = 16.2$ cm with the sugar added in initial background fluid.

The results for two groups of experiments were shown in Figure 4 and Figure 5 respectively. It is worth noticing that our measurement reflects only the change of the torque instead of torque itself. By setting the starting torque at time zero to be zero dyne·cm, we obtain the change of torque before 7000 seconds with different four lines shows four rounds in this group. Time zero is defined to be the time when the torsion spring reaches the equilibrium as we observed from the movement of the light spot. Figure 4b is the zoomed-in version of Figure 4a which we set the time window starting from 1000 seconds to 2000 seconds. Figure 4c plots the Fourier transform of the torque series from 0 seconds to 7000 seconds for each round in the group. As for ingredients, in every 300 grams of water, we add 40 grams sugar + 20 grams syrup, the ratio of sugar and syrup is the same across the fluid and the candy.



(a) Torque in the process of dissolve (b) Torque in from 1000s to 2000s (c) Spectral of torque

Figure 4: Measured torque in our four identical setups



(a) Torque in the process of dissolve (b) Torque in from 1000s to 2000s (c) Spectral of torque

Figure 5: Measured torque by varying parameters of the inner cylinder

With fixed parameters, the change of torque over 7000 seconds can be quite different. However, we can always observe the oscillation with the similar period as shown in the spectral(Figure 4c). By decreasing the size of the cylinder, we did not see any obvious changes. However, when the concentration of the candy in the initial fluid increases, a significant decrease in oscillation amplitude occurs as shown in Figure 5b.

5 Summary

In this project, we present the possibility of measuring the torque caused by a dissolving body using Cavendish's method. We start by introducing project setup in Section 1 and 2. Then we move on to present the measurement method for change of torque in Section 3. Finally, we show our experiment results in Section 4, as such, we demonstrate that:

1. During a long time scale(7000 seconds), the change of torque could be quite different but the oscillation period keeps constant.
2. Changing cylinder size does not influence the change of torque while adding sugar in initial fluid decreases the amplitude of oscillation.

Due to time limitations, we are not able to change the initial fluid with different concentrations which could be a possible improvement for future work.

6 Acknowledgment

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